Quiz 3 (6.6i, 6.1ii, 6.2ii) (20 points)

You may use computers to check, but you most show the calculus you did, including all steps to find the following.

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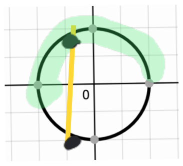
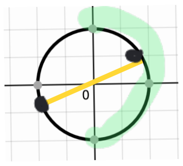
(1). Find the exact values of the following: (4 points)

(Note: Unless otherwise stated, it is always expected that answers are exact values rather than calculator approximations)

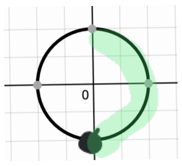


(a) $\cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$

(b) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \boxed{\frac{\pi}{6}}$



Can check this with calculator

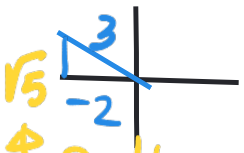


(c) $\sin^{-1}(-1) = \boxed{\frac{-\pi}{2}}$

(d) $\sin\left(\underbrace{\cos^{-1}\left(-\frac{2}{3}\right)}\right) = \sin\theta = \boxed{\frac{\sqrt{5}}{3}}$

Note: It cannot be $\frac{3\pi}{2}$ since $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\theta = \cos^{-1}\left(-\frac{2}{3}\right) \Rightarrow \begin{cases} \cos\theta = -2/3 \\ \text{and} \\ 0 \le \theta \le \pi \end{cases}$



pythagorean

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(2) Given $f(x) = x^3 + x + 3$ find $\left(f^{-1}\right)'(3)$ (3 points)

Show steps clearly, with explanation

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))}$$

$$= \frac{1}{3(0)^2 + 1}$$

$$= \boxed{1}$$

Find needed pieces

$$\begin{cases} f'(x) = 3x^2 + 1 \\ f^{-1}(3) = 0 \end{cases} \xrightarrow{} \text{how?}$$

$(3, f^{-1}(3))$

becomes $(f^{-1}(3), 3)$ for f

So when does $f = 3$

$3 = x^3 + x + 3$

$\Rightarrow x = 0$

So $(0,3)$ on f

$(3,0)$ on f^{-1}

$f^{-1}(3) = 0$

(3) Find $f'(x)$ if $f(x) = e^{3\cos(x^2)}$ (3 points)

$$f'(x) = e^{3\cos(x^2)} \frac{d}{dx}\left(3\cos(x^2)\right)$$

chain rule

notation

$$= e^{3\cos(x^2)}\left(-3\sin(x^2)\frac{d}{dx}(x^2)\right)$$

chain rule again

$$\boxed{f'(x) = -6x\sin(x^2)e^{3\cos(x^2)}}$$

These can be checked on the computer

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(4) Evaluate $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ (3 points)

Watch the notation. Do not write x's limits with du.

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx \Rightarrow 2du = \frac{1}{\sqrt{x}}dx$

changing limits

$u = \sqrt{x}$

$x = 4 \Rightarrow u = 2$

$x = 1 \Rightarrow u = 1$

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = \int_1^4 e^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}}dx$$

$$= \int_1^2 e^u \, 2du = 2e^u\Big]_1^2 = \boxed{2(e^2 - e)}$$

(5) Given $f(x) = xe^x$, give a thorough answer to the following, showing all work. Then sketch the graph.

Note We will learn in a later section $\lim_{x\to-\infty} xe^x = 0$ Use this information to help with your graph

(8 points)

(a) domain $(-\infty,\infty)$

(b) x intercept(s) = 0

$xe^x = 0 \Rightarrow x = 0$

(e) local extrema local min at $(-1, \frac{-1}{e})$

(f) Inflection points. at $(-2, \frac{-2}{e^2})$

$f'(x) = e^x + xe^x$

$= e^x(1+x)$

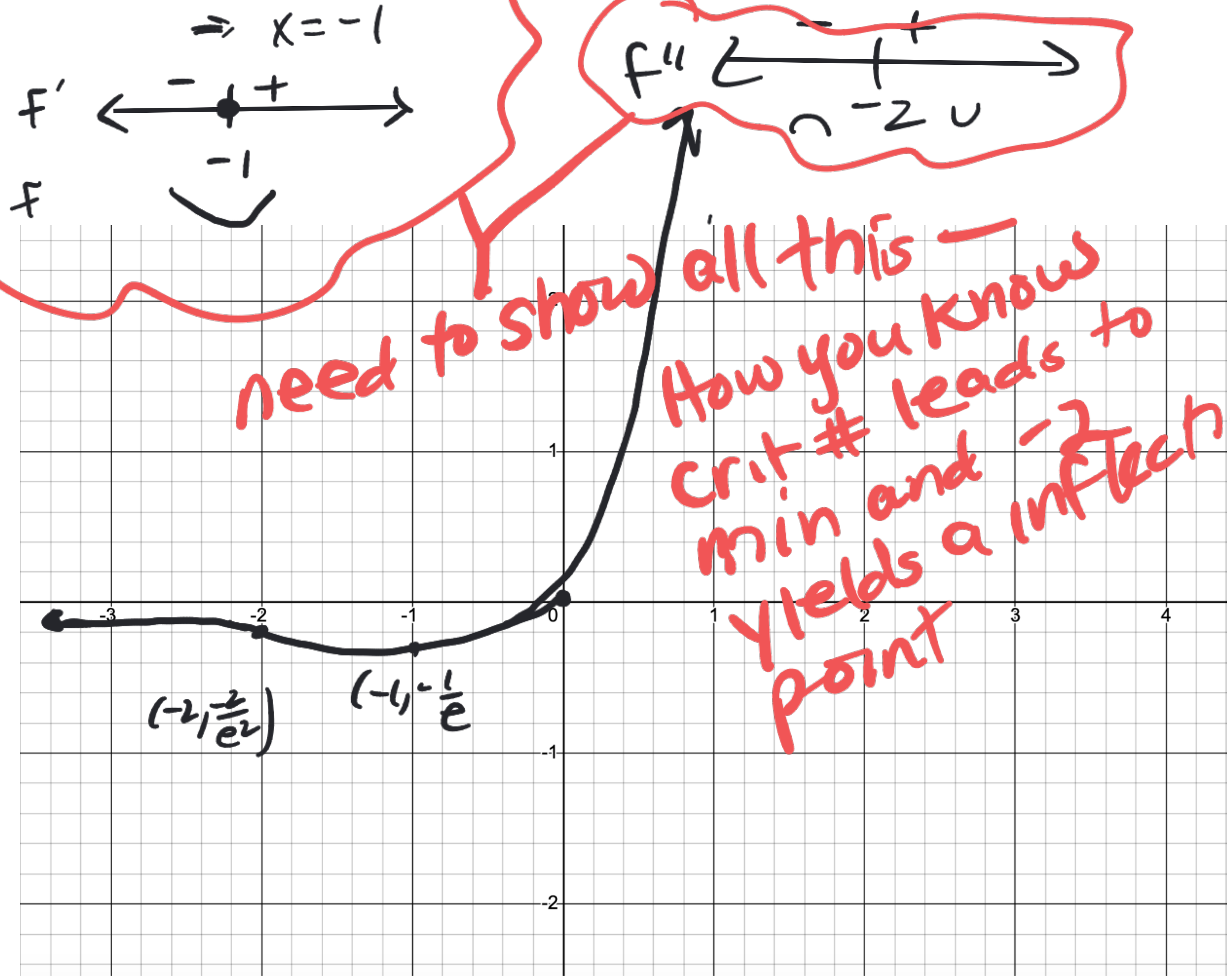
$f''(x) = e^x + e^x + xe^x$

$= e^x(2+x)$

$f''(x) = 0$ when $x = -2$

Crit #s $e^x(1+x) = 0$

$\Rightarrow x = -1$



need to show all this — How you know crit # leads to min and -2 yields a inflection point